

The Math Multi-tool

The Arithmetic Rack - The Rekenrek - The Bead Rack Many names - - Many uses

The arithmetic rack is a calculating frame consisting of two rows of ten beads with two sets of five beads (2 colors) in each row. It was developed by Adri Treffers, a researcher at the Freudenthal Institute in the Netherlands. Experience with the arithmetic rack allows young children to build on their innate ability to subitize small groups and to see five as a unit. When five can be subitized as a whole, it can be used to support understanding of 6 as $5+1$, 8 as $5+3$, or 4 as $5-1$. The arithmetic rack also supports the strategies of using doubles and near doubles to learn basic facts, such as $6+7=6+6+1$, and making tens, such as $9+6=10+5$.

Minilessons for Early Addition and Subtraction
Catherine Twomey Fosnot and Willem Uittenbogaard

Subitizing

Decompose/compose Numbers - Part-whole construction of number

Flashing - Briefly displaying and then concealing the visual materials

Using the 5 and 10 Structure

5 on bottom
5 on top, 5 on bottom
6 on bottom
8 on bottom
7 on bottom
2 on top, 5 on bottom
10 on top

p. 25

10 on top, 7 on bottom
9 on top, 8 on bottom
10 on top, 2 on bottom
9 on top, 3 on bottom
9 on top, 6 on bottom
3 on top, 8 on bottom
8 on top, 7 on bottom

p. 33

Using Doubles or Pair-Wise

3 on top, 3 on bottom
5 on top, 5 on bottom
5 on top, 6 on bottom
8 on top, 8 on bottom
7 on top, 8 on bottom
6 on top, 6 on bottom
7 on top, 6 on bottom

p. 36

7 on top, 7 on bottom
8 on top, 6 on bottom
9 on top, 5 on bottom
7 on top, 7 on bottom
9 on top, 7 on bottom
7 on top, 5 on bottom
6 on top, 8 on bottom

p. 41

Screening

- Distance the Setting and lead to missing addend problems
Concealing part or all of the elements within a setting

Using the 5 and 10 structure

Show 6 on top, screen bottom (10)
Show 4 on bottom, screen top (11)
Show 7 on top, screen bottom (10)
Show 5 on bottom, screen top (11)
Show 5 on top, screen bottom (11)
Show 4 on top, screen bottom (10)

Using Doubles or Pair-Wise

Show 3 on top, screen bottom (6)
Show 3 on top, screen bottom (7)
Show 4 on top, screen bottom (8)
Show 4 on top, screen bottom (9)
Show 8 on top, screen bottom (16)
Show 8 on top, screen bottom (15)

Multiplication - Repeated Addition and Arrays

(Use 100 Bead Rack - 10 rows of 10 with two sets of 5 beads (2 colors) in each row.)

4x7 or an array of 4 rows with 7 beads in each row

Repeated addition or sequence counting

Count by 7 four times or count by 4 seven times.

Count by 5 four times=20 and count by 2 four times=8. $20+8=28$

Decompose 7 as 5+2

See $4 \times 5 = 20$ and $4 \times 2 = 8$ So, 4×7 is $20+8=28$

This leads into the open array model for multi-digit multiplication

Using the Double-Double Strategy for multiplying by 4

See 7 doubled is 14 and 14 double is 28

Decompose then double-double - $5 \times 2 = 10$ $10 \times 2 = 20$. $2 \times 2 = 4$ $4 \times 2 = 8$ So, $20+8=28$

8x8 or an array of 8 rows with 8 beads in each row

Repeated addition or sequence counting

Count by 8 eight times.

Count 5 eight times=40 and count 3 eight times=24. $40+24=64$

Decompose 8 as 5+3

See $8 \times 5 = 40$ (or $4 \times 5 \times 2$) and $3 \times 8 = 24$ (or $3 \times 4 \times 2$). So, 8×8 is $40+24=64$

Using the Double-Double-Double Strategy for multiplying by 8

8 doubled is 16. 16 doubled is 32. 32 doubled is 64.

Decompose then double-double-double

$5 \times 2 = 10$ $10 \times 2 = 20$ $20 \times 2 = 40$. $3 \times 2 = 6$ $6 \times 2 = 12$ $12 \times 2 = 24$ So, $40+24=64$

With all of these strands/minilessons/techniques, the most important aspect is the discussion that takes place. It is extremely important to ask questions such as:

How did you see that?

How did you know that was "8"?

Do you agree with "name"? Why/why not?

Did anyone see it differently?

Ask one student to explain another student's strategy.

How did you know what was missing? (screening)